Math 110

Winter 2021

Lecture 12

Consider a binomial prob. dist with $n=250$ and $p=.8$

Find

1) $q=1-p=\boxed{.2}$

2) $\mu=np=\boxed{200}$

3) $\sigma^2=npq=40$

4) $\sigma=\sqrt{\sigma^2}=\sqrt{40}\approx\boxed{6.325}$

Round μ & σ to a whole #, find

$\boxed{\mu=200}$ $\boxed{\sigma=6}$

5) 68% Range

$\mu\pm\sigma=200\pm6\Rightarrow\boxed{194 \text{ to } 206}$

6) Usual Range

95% Range

$\mu\pm2\sigma=200\pm2(6)$

$\boxed{188 \text{ to } 212}$

Let x be # of successes, find

7) $P(x=185)$

$=\text{binompdf}(250, .8, 185)$

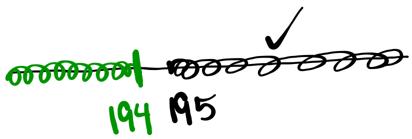
$=\boxed{.004}$

8) $P(x\le 210)$

$=\text{binomcdf}(250, .8, 210)$

$=\boxed{.955}$

9) $P(x \geq 195)$



$= 1 - P(x \leq 194)$

$= 1 - \text{binomcdf}(250, .8, 194)$

$= \boxed{.809}$

10) $P(190 \leq x \leq 215)$ Reduce by 1

$= \text{binomcdf}(250, .8, 215)$

$- \text{binomcdf}(250, .8, 189)$

$= \boxed{.944}$

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180 randomly selected voters.

Prob. that any voter supports tougher gun law is .6.

Find

1) $n = 180$ 2) $p = .6$ 3) $q = 1 - p = \boxed{.4}$ 4) $\mu = np = \boxed{108}$

5) $\sigma^2 = npq = \boxed{43.2}$ 6) $\sigma = \sqrt{\sigma^2} = \boxed{6.573}$

Round μ & σ to a whole #, then find $\boxed{\mu = 108}$ $\boxed{\sigma = 7}$

7) 68% Range

$\mu \pm \sigma = 108 \pm 7 \Rightarrow \boxed{101 \text{ to } 115}$

8) Usual Range

95% Range

$\mu \pm 2\sigma = 108 \pm 2(7)$

$\Rightarrow \boxed{94 \text{ to } 122}$

9) P(exactly 100 of these voters support such law)

$= P(x = 100) = \text{binompdf}(180, .6, 100) = \boxed{.029}$

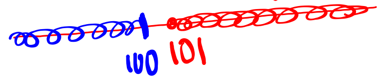
10) P(fewer than 110 of these voters support such law)

$= P(x < 110) = P(x \leq 109) = \text{binomcdf}(180, .6, 109)$

$= \boxed{.588}$

11) P(more than 100 of these voters support such law)

$= P(x > 100) = P(x \geq 101) = 1 - \text{binomcdf}(180, .6, 100)$



$= \boxed{.873}$

12) P(between 95 and 120, inclusive, of these voters support such law)

$= P(95 \leq x \leq 120) = \text{binomcdf}(180, .6, 120) - \text{binomcdf}(180, .6, 94)$

Reduce by 1

$= \boxed{.952}$

Geometric Prob. Dist SG 18

1) like binomial prob. dist, events are independent but no fixed n.

2) Trials are repeated until success happens.

3) Prob. of success $= p$, Prob. of failure $= q$
$p + q = 1$, $q = 1 - p$
p remains the same for each trials.

$P(x) = p \cdot q^{x-1}$

$\mu = \frac{1}{p}$, $\sigma^2 = \frac{q}{p^2}$, $\sigma = \sqrt{\sigma^2}$

ex: Consider a geometric prob. dist with $p = .4$

$q = 1 - p$ $\quad \mu = \frac{1}{p} = \frac{1}{.4}$ $\quad \sigma^2 = \frac{q}{p^2} = \frac{.6}{.4^2}$

$\boxed{q = .6}$ $\quad \boxed{\mu = 2.5}$ $\quad \boxed{\sigma^2 = 3.75}$

$\sigma = \sqrt{\sigma^2} = \sqrt{3.75} = \boxed{1.936}$

$P(x = 4) = (.4)(.6)^{4-1} = (.4)(.6)^3 = \boxed{.086}$

$P(x) = p \cdot q^{x-1}$ using TI 2nd VARS geometpdf

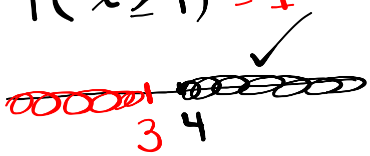
$P(x=4) = \text{geometpdf}(.4, 4) = \boxed{.086}$

$P(x \le 5) = P(X=5) + P(X=4) + P(X=3) + P(X=2) + P(X=1)$

$= \text{geometcdf}(.4, 5) = \boxed{.922}$

$P(x \ge 4) = 1 - P(x \le 3) = 1 - \text{geometcdf}(.4, 3)$

$= \boxed{.216}$



Kabeer makes a shot with prob. of .6

1) $p = .6$

2) $q = .4$

3) $\mu = \frac{1}{p} = \frac{1}{.6} = \boxed{1.667}$

4) $\sigma^2 = \frac{q}{p^2} = \frac{.4}{.6^2} = \boxed{1.111}$

5) $\sigma = \sqrt{\sigma^2} = \boxed{1.054}$

6) P(he makes first shot on 3rd trial)

$= P(x=3) = \text{geomet pdf}(.6, 3) = \boxed{.096}$

7) P(he makes first shot before the 5th trial)

$= P(x < 5) = P(x \le 4) = \text{geometcdf}(.6, 4)$

$= \boxed{.974}$

8) P(he makes first shot after the 5th trial)

$= P(x > 5) = P(x \ge 6) = 1 - P(x \le 5)$

$= 1 - \text{geometcdf}(.6, 5) = \boxed{.010}$

Poisson Prob. Dist.

1) It takes place on a fixed interval.

2) the mean number of Success in that interval is μ.

x is # of Successes

$$P(x) = \frac{\mu^x}{x!} e^{-\mu}, \quad e \approx 2.7183$$

$x = 0, 1, 2, 3, \ldots$

$\sigma^2 = \mu \qquad \sigma = \sqrt{\sigma^2}$

$\lambda = \mu$

$P(x = a) = \text{poissonpdf}(\mu, a)$

$P(x \leq a) = \text{Poissoncdf}(\mu, a)$

$P(x \geq a) = 1 - \text{Poissoncdf}(\mu, a-1)$

Gustavo gets in average 20 text messeses in 10 hrs. Every hrs, in average, he gets 2 text messages.

$\mu = 2$ Fixed Interval $\Rightarrow$ Every hour.

$\mu = 2 \qquad \sigma^2 = \mu \quad \sigma^2 = 2 \qquad \sigma = \sqrt{\sigma^2} = \sqrt{2} \approx 1.414$

P(he gets 3 text MSG) $= P(x = 3) = \text{Poisson Pdf}(2, 3) = \boxed{.180}$

P(he gets at most 2 MSG) $= P(x \leq 2) = \text{Poisson Cdf}(2, 2) = \boxed{.677}$

P(he gets at least 1 MSG) =

$P(x \geq 1) = 1 - P(x = 0) = 1 - \text{PoissonCdf}(2, 0) = \boxed{.865}$

At a Fundraising event, You buy a ticket For \$1, choose any 3 numbers from 1 to 25.
Fundraiser draw 3 numbers,
If You have all 3 ⇒ You get \$100
If You have only 2 ⇒ You get \$10
If You have only 1 ⇒ You get \$1
otherwise, You get nothing. Find expected Value Per ticket.

# win.	\$Net	P(Net)
3	1 − 100	$\frac{{}_3C_3 \cdot {}_{22}C_0}{{}_{25}C_3} = \frac{1}{2300}$
2	1 − 10	$\frac{{}_3C_2 \cdot {}_{22}C_1}{{}_{25}C_3} = \frac{66}{2300}$
1	1 − 1	$\frac{{}_3C_1 \cdot {}_{22}C_2}{{}_{25}C_3} = \frac{693}{2300}$
0	1 − 0	$\frac{{}_3C_0 \cdot {}_{22}C_3}{{}_{25}C_3} = \frac{1540}{2300}$

\$Net → L1
P(\$Net) → L2

Use L1 & L2 to Find Expected Value $= \mu = \bar{x}$
.368
≈ 37 ¢/TKT

Pay me \$5, Draw one Card from a Standard deck of playing Cards.
If You draw an Ace → I give You \$25
If You draw a face → I give You \$15
any other card, I give You nothing.
Find expected value per bet for the house.

Net gain	P(Net gain)	
5 − 25	$\frac{4}{52}$	Ace
5 − 15	$\frac{12}{52}$	face
5 − 0	$\frac{36}{52}$	Any other Card

Net gain → L1, P(Net gain) → L2

Expected Value Per bet $= \mu = \bar{x} = -.385$

I lose ≈ 39 ¢ Per bet.

I lose \$ in this Process.

Class QZ 7

$P(A) = .7$ $P(B) = .5$ $P(A \text{ and } B) = .4$

1) Venn Diagram

2) $P(A \text{ or } B)$

3) $P(A|B)$